



## Stages in Addition



### Mental Methods and calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

### Addition – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities.



They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

They will find one more than a given number up to 20 and beyond. In practical activities and through discussion they will begin to use the vocabulary involved in addition.



'You have five apples and I have three apples. How many apples altogether?'

### Addition – Year 4

-  Add numbers with up to 4 digits using the formal written method of columnar addition where appropriate
-  Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Continue to teach the use of empty number lines with three and 4-digit numbers, as appropriate.

Further develop the formal written method of addition, with 3-digit numbers.

Revisit the expanded method first, if necessary:

$$177 + 146 = 323$$

$$177 + 146$$

$$13 (7+6)$$

$$110 (70+40)$$

$$200 (100 + 100)$$



$$323$$

\*Our children know this as the Partitioning (PA) method

This will lead into the formal written method without carrying and then with carrying with up to 4-digit numbers.



### Addition – Year 5

-  Add whole numbers with more than 4-digits, including using a formal written method (columnar addition)
-  Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Continue to teach the use of empty number lines with larger numbers (and decimals), as appropriate.

Continue to develop the formal written method for addition with larger numbers (and decimal numbers) and with the addition of three or more numbers:

$$21,848 + 1,523 = 23,371$$

$$\begin{array}{r} 21848 \\ + 1523 \\ \hline 23371 \end{array}$$

Continue to use the language of place value to ensure understanding. Ensure that the digits that have been 'carried' are recorded under the line in the correct column.

Use the formal written method for the addition of decimal numbers:  
£254.75 + £233.82 = £488.57

$$\begin{array}{r} 254.75 \\ + 233.82 \\ \hline 488.57 \end{array}$$

Ensure that the decimal points line up.

### Addition – Year 6

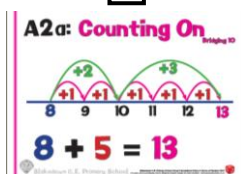
No objectives have been included in the programmes of study explicitly related to written methods for addition in Year 6. However, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance and exemplification of methods).

Our aim is that by the end of Year 6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Addition – Year 1

- Given a number, identify one more and one less
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign
- Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most and least
- Add 1-digit and 2-digit numbers within 20, including zero
- Represent and use number bonds and related subtraction facts within 20
- Solve 1-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as:

$$9 = \square + 4$$



## Addition – Year 2

- Add numbers using concrete objects, pictorial representations, and mentally, including:

A 2-digit number and ones

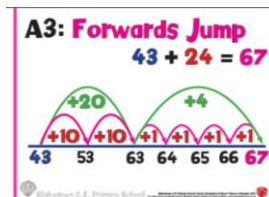
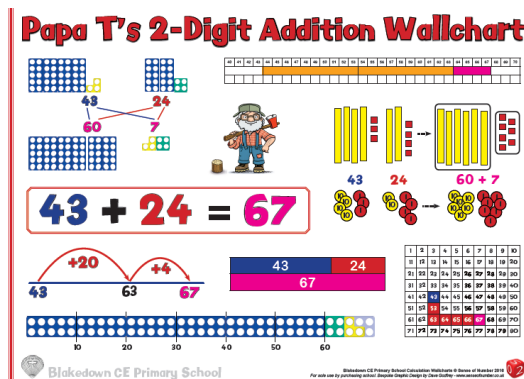
A 2-digit number and tens

Two 2-digit numbers

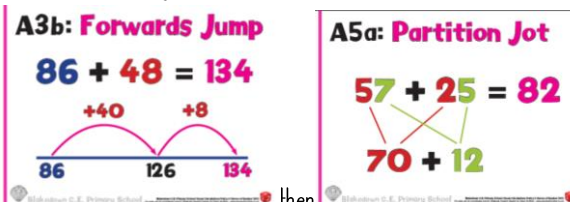
Three 1-digit numbers

- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures

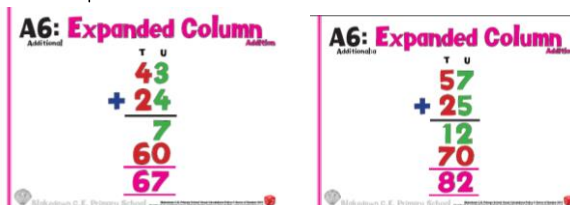
- Applying their increasing knowledge of mental and written methods



- Counting on in tens
- Counting on in tens efficiently
- Partitioning



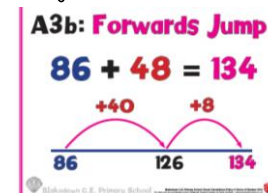
- Expanded written method



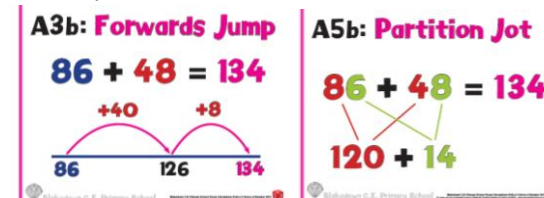
## Addition – Year 3

- Add numbers with up to 3-digits, using a formal written method of columnar addition
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

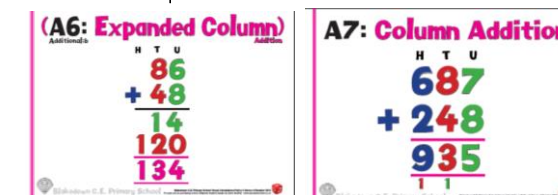
Further develop the use of the empty number line with calculations that bridge 100:



Further develop the partitioning method with calculations that also bridge 100:



Then use the expanded column method:





## Subtraction — Year 4

- Subtract numbers with up to 4-digits using the formal written method of columnar subtraction where appropriate
- Solve addition and subtraction 2-step problems in contexts, deciding which operations and methods to use and why

Continue to teach the use of empty number lines with 3 and 4 – digit numbers, as appropriate

Continue to develop the formal written method of subtraction by revisiting the expanded method first, if necessary. Continue to use base ten materials to support understanding.



Without exchanging then... with exchanging

When children are confident, develop 4-digit numbers and decimal numbers (in the context of money and measures)



## Subtraction — Year 5

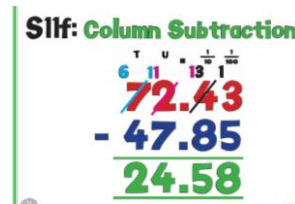
- Subtract whole numbers with more than 4-digits, including using a formal written method (columnar subtraction)
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.

Continue to develop the formal written method for subtraction with 3 and 4-digit numbers (See Y4 guidance), returning to an expanded method and using base ten materials, if necessary.

$$503 - 278 = 225$$

Introduce subtraction of decimals, initially in the context of money and measures.



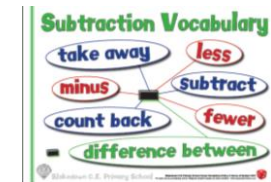
Ensure the decimal point is lined up accurately.

## Subtraction — Year 6

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in Year 6. However, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance and exemplification of methods).

Our aim is that by the end of Year 6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Stages in Subtraction



### Mental Methods and calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts.

Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

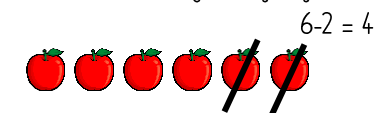
### Subtraction — Early Stages (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.



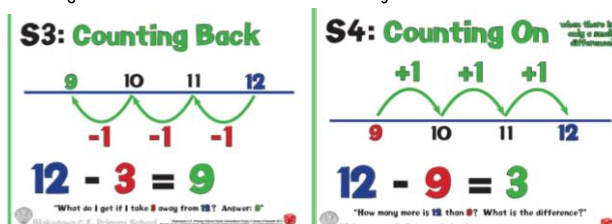
Children will begin to count back from a given number.

## Subtraction — Year 1

- Given a number, identify one more and one less
- Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign
- Represent and use number bonds and relate subtraction facts within 20
- Subtract 1-digit numbers and 2-digit numbers within 20, including zero
- Solve missing number problems e.g.  
 $20 - \square = 15$

Children will continue to practise counting back from a given number.

Initially use a number line to count back for subtraction:



Put your finger on number 9. Count back 3 and what do we have?

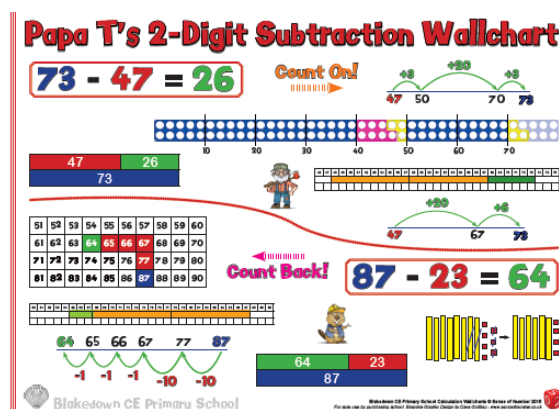
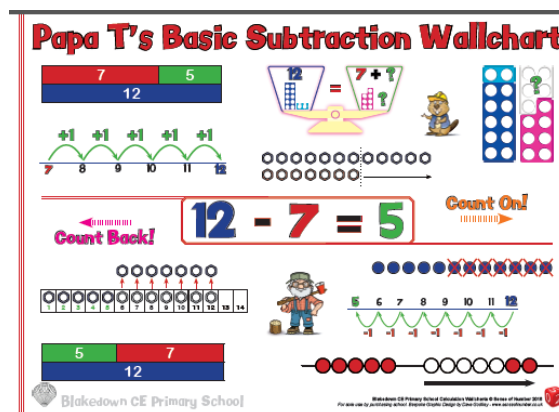
We can count on and count back to find the difference within a number sentence. But counting on only really helps when there is a small difference.

## Subtraction — Year 2

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - A 2-digit number and ones
  - A 2-digit number and tens
  - Two 2-digit numbers

- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Solve problems with addition and subtraction:
  - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - Applying their increasing knowledge of mental and written methods

Counting back using an empty number line within 100, in ones and tens.



Introduce complementary addition to find differences (only for small differences). The use of models is extremely important here to understand the idea of 'difference'.

Count up from the smallest number to the largest to find the difference

$$12 - 8 = 4$$

## Subtraction — Year 3

- Subtract numbers with up to 3-digits, using formal written method of columnar subtraction
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Further develop the use of the empty number line with calculations that bridge 100:

$$126 - 45 = 81$$

Extend with larger numbers by counting back...

$$216 - 27 = 189$$

...and by counting on to find the difference (small difference):

$$231 - 198 = 33$$

## MS2: Counting On

$$75 - 47 = 28$$



## S10: Expanded Subtraction

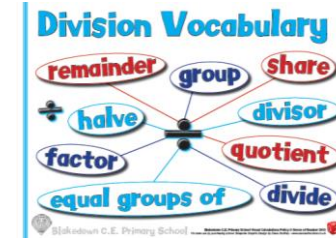
$$723 - 356 = 367$$







## Stages in Division



### Mental Methods and calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

### Division — Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities through discussion they will begin to solve problems involving halving and sharing.



Share the apples between two people.

'Half of the apples for you and half of the apples for me.'

## Division — Year 4

- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying by three numbers
- Divide 2-digit and 3-digit numbers by a 1-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

$$32 \div 8 = 4$$

Continue to use the formal written layout for division using multiplication tables that they know:

'How many eights are there in thirty-two?'

Continue using the **formal written layout, introducing remainders:**

NB Remainders are not specifically referred to until Year 5 in the National Curriculum

However, this may be an appropriate point to introduce them using familiar multiplication facts.

## Division — Year 5

- Multiply and divide numbers up to 4-digits by an 1-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Continue to practise the **formal written method of short division** with whole number answers...

$$184 \div 8 = 23$$

...and with remainders

The remainder can also be expressed as a fraction, 4/6 (the remainder divided by the divisor)

$$394 \div 6 = 65 \frac{4}{6}$$

Continue to practise, develop and extend the formal written method of short division, with and without remainders. Extend to decimal remainders. Interpret and express remainders according to the context.

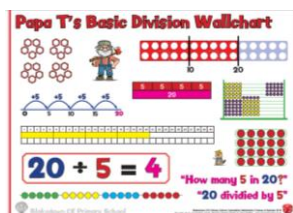
## Division — Year 6

- Solve problems involving addition, subtraction, multiplication and division
- Multiply and divide numbers up to 4-digits by a 2-digit number using the formal written method of **short division** where appropriate, interpreting remainders according to the context (non-statutory guidance)
- Divide numbers up to 4-digits by a 2-digit whole number using the formal written method of **long division**, and interpret remainders as whole number remainder, fractions, or by rounding, and decimals as appropriate for the context (non-statutory guidance)

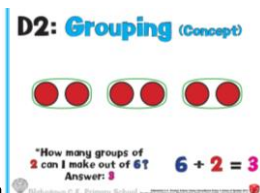
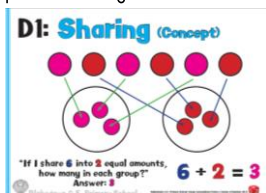
## Division — Year 1

- Solve 1-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher
- Count in multiple of 2s, 5s and 10s (to the 10<sup>th</sup> multiple)

Children will start with practical sharing using a variety of resources.



They will share objects into equal groups in a variety of situations. Children will move forward from **sharing** to **grouping** in a practical way



...then

'Put 6 crayons into groups of 2. How many pots do we need?'

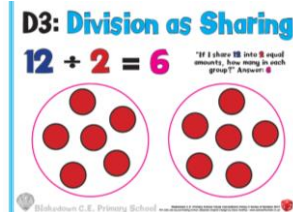
Use arrays to support early division (see image above)

'How many counters altogether? How many groups of two?'

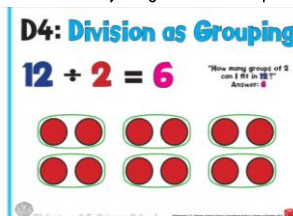
## Division — Year 2

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for division within the multiplication tables they know and write them using the division ( $\div$ ) and equals (=) signs
- Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts

Sharing and grouping:

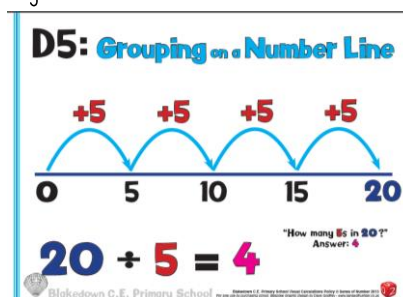


'12 crayons shared equally between 2 pots' (sharing)



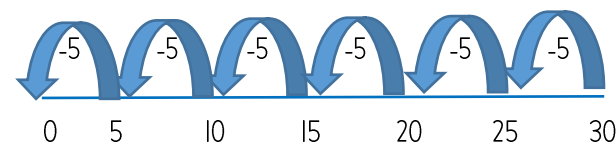
'We have 12 crayons and put 6 crayons in each pot. How many pots do we need?' (grouping)

Grouping using a number line:



Also jump back to make the link with repeated subtraction:

$$30 \div 5 = 6$$



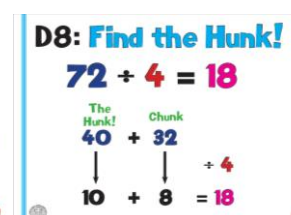
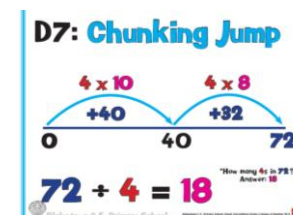
'How many groups of 5?'

## Division — Year 3

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for 2-digit numbers divided by 1-digit numbers, using mental methods and progressing to a formal written method
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which objects are connected to objects.

Continue to use practical resources, pictures diagrams, number lines, arrays and the  $\div$  sign to record, using multiples that they know, as appropriate.

Using an empty number line to count forwards in jumps... then find the 'Hunk' when moving towards a more formal written method.



Introduce the formal layout using multiplication/division facts that the children know:

$$24 \div 3 = 8$$

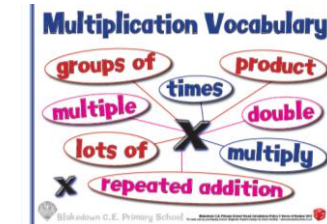
This can also be recorded as ...

$$\begin{array}{r} 08 \\ 3 \overline{) 24} \end{array}$$

'Twenty four divided by three equals eight'



## Stages in Multiplication



### Mental Methods and calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

### Multiplication – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities through discussion they will begin to solve problems involving doubling.



'Three apples for you and three apples for me. How many apples altogether?'

## Multiplication – Year 5

Multiply numbers up to 4-digits by a 1 or 2-digit number using a **formal written method**, including long multiplication for 2-digit numbers

Build on the work covered in Year 4 with the **formal method** of **short multiplication** (2-digit number multiplied by a 1-digit number). When children are confident introduce multiplication by a 2-digit number. If necessary, return to the grid method and/or expanded method first.

Grid method (2-digit number multiplied by a 2-digit)

**M8: Grid Method**  
Long Multiplication

$$43 \times 65 = 2795$$

x	40	3
60	2400	180
5	200	15

$2400 + 180 + 200 + 15 = 2795$

**M9: Long Multiplication**

$$\begin{array}{r} 43 \\ \times 65 \\ \hline 215 \quad (5 \times 43) \\ + 2580 \quad (60 \times 43) \\ \hline 2795 \end{array}$$

This can then advance to a short multiplication method if children feel confident to do so.

## Multiplication – Year 6

Multiply multi-digit numbers (including decimals) up to 4-digit by 2-digit whole numbers

Continue to practise and develop the formal short multiplication method with larger numbers and decimals throughout

**M9g: Long Multiplication**

$$\begin{array}{r} 3786 \\ \times 48 \\ \hline 30288 \quad (8 \times 3786) \\ + 151440 \quad (60 \times 3786) \\ \hline 181728 \end{array}$$

**M9f: Long Multiplication**

$$\begin{array}{r} 24.3 \\ \times 2.5 \\ \hline 12.15 \quad (0.5 \times 24.3) \\ + 48.60 \quad (2 \times 24.3) \\ \hline 60.75 \end{array}$$

**M8f: Grid Method**  
Long Multiplication

$$24.3 \times 2.5 = 60.75$$

x	20	4	0.3
2	40	8	0.6
0.5	10	2	0.15

$48.6 + 12.15 = 60.75$

**M9e: Column Multiplication**

$$\begin{array}{r} 24.3 \\ \times 2.5 \\ \hline 12.15 \\ + 48.60 \\ \hline 60.75 \end{array}$$

## Multiplication – Year 4

Recall multiplication facts for multiplication tables up to 12 x 12

Multiply 2-digit and 3-digit numbers by a 1-digit number using **formal written layout**

Solve problems involving multiplication and adding, including using the distributive law to multiply 2-digit numbers by 1-digit, integer scaling problems and harder correspondence problems such as 'n' objects are connected to 'x' objects.

Continue to use empty number lines, as appropriate

Further develop the grid method for 2-digit numbers multiplied by a 1-digit number

**Expanded short multiplication** (2-digit number by a 1-digit number)

Refine the recording in preparation for **formal short multiplication**:

**M6: Expanded Column**  
Additional

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \quad (6 \times 3) \\ 240 \quad (6 \times 40) \\ \hline 258 \end{array}$$


This leads to short multiplication (formal method) of a 2-digit number multiplied by a 1-digit number.


**M7: Column Multiplication**  
Additional

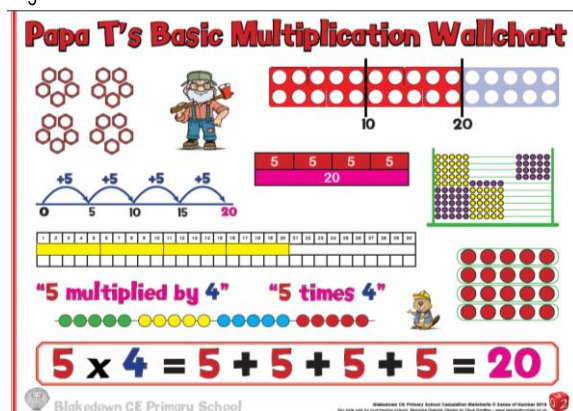
$$\begin{array}{r} 43 \\ \times 6 \\ \hline 258 \end{array}$$



## Multiplication — Year 1

 Solve 1-step problems involving multiplication by calculating the answers using concrete objects, pictorial representations and arrays with the support of the teacher

 Count in multiples of 2s, 5s and 10s (to the 10<sup>th</sup> multiple)  
Children will count repeated groups of the same size practical object



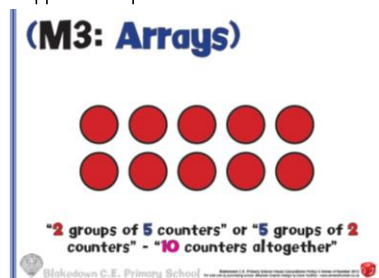
They will solve practical problems that involve combining groups of 2, 5 or 10 e.g. Socks, fingers and cubes.




Six pairs of socks altogether


How many socks are there altogether? 2, 4, 6, 8, 10, 12


Use arrays to support multiplication



## Multiplication — Year 2

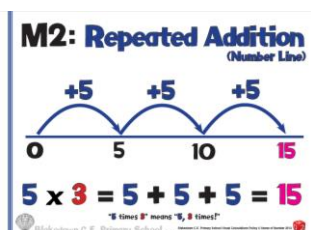
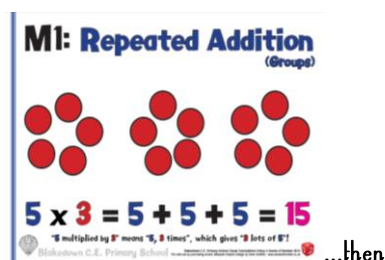
 Recall and use multiplication facts for the 2, 5 and 10 multiplication tables including recognising odd and even numbers

 Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs

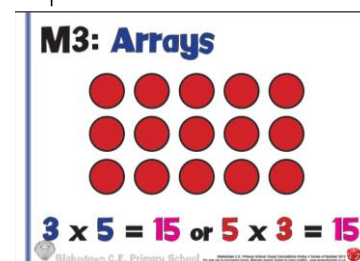
 Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

Show that multiplication of two numbers can be done in any order (commutative)


Repeated addition:





Use arrays to support the children's understanding of repeated addition into multiplication:



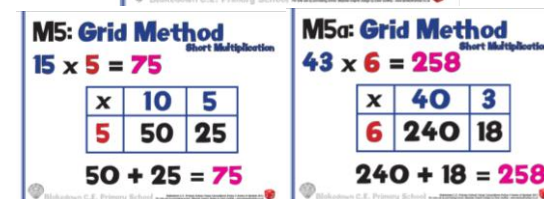
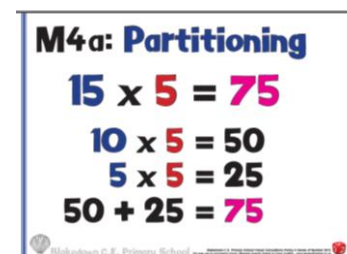
## Multiplication — Year 3

 Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)

 Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental methods and progressing to a formal written method

 Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems which 'n' objects are connected to 'x' objects

Continue to use number lines and arrays to support multiplication, as appropriate:



Then move on to an expanded short multiplication to aid the transition between their accurate understanding and a formal written response:

